Question 15 (8 marks)

The probability that an adult Ragdoll cat of weight kg is a female can be modelled by the logistic equation

(a) Calculate the probability that a cat of weight kg is a female. (1 mark)

(b) Sketch the graph of this model on the axes below. (2 marks)

<EFOFEX>
id:fxd{db64dd9b-f3cd-453b-9c91-e99e7a68314a}

FXData:

</EFOFEX>

(c) The logistic equation can be written in the form . State the value of and the value of . (2 marks)

(d) The sensitivity of the model is defined as the absolute value of the change in for a one-gram increase in the weight of a cat. Determine the maximum value of .

(3 marks)

Question 15 (8 marks)

The probability that an adult Ragdoll cat of weight kg is a female can be modelled by the logistic equation

(a) Calculate the probability that a cat of weight kg is a female. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

(b) Sketch the graph of this model on the axes below. (2 marks)

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ endpoints and logistic 'S' curve  ü point of inflection close to |

<EFOFEX>
id:fxd{bf423b89-c5d7-4b1f-a813-46770021a79a}

FXData:

</EFOFEX>

(c) The logistic equation can be written in the form . State the value of and the value of . (2 marks)

|  |
| --- |
| Solution |
| is limiting value as and so .  From defining rule, . |
| Specific behaviours |
| ✓ value of  ü value of |

(d) The sensitivity of the model is defined as the absolute value of the change in for a one-gram increase in the weight of a cat. Determine the maximum value of .

(3 marks)

|  |
| --- |
| Solution |
| changing fastest at point of inflection, when .  This rate is in kg, hence . |
| Specific behaviours |
| ✓ indicates where the rate of change is greatest  ü obtains value of at point of inflection  ü correct maximum value of |

Question 18 (7 marks)

A small body is moving in a straight line so that seconds after leaving fixed point its velocity is cm/s and its acceleration cm/s2, where and are constants.

Initially the body is at rest at and its acceleration is cm/s2.

seconds later, its velocity is m/s and its acceleration is cm/s2.

(a) Show that . (3 marks)

(b) Determine the exact value of . (4 marks)

Question 18 (7 marks)

A small body is moving in a straight line so that seconds after leaving fixed point its velocity is cm/s and its acceleration cm/s2, where and are constants.

Initially the body is at rest at and its acceleration is cm/s2.

seconds later, its velocity is m/s and its acceleration is m/s2.

(a) Show that . (3 marks)

|  |
| --- |
| Solution |
| When and when . Using :  Hence |
| Specific behaviours |
| ✓ obtains value of  ü obtains value of  ü uses to obtain equation |

(b) Determine the exact value of . (4 marks)

|  |
| --- |
| Solution |
| When and so  When and so |
| Specific behaviours |
| ✓ separates variables  ü obtains correct antiderivative  ü evaluates constant of integration  ü obtains exact value of |

Question 13 (8 marks)

A person who weighs kg begins a specialist diet so that their rate of weight loss can be modelled by

where is the persons weight in kilograms and is the number of days since the diet began.

After week the person had lost a total of kg.

(a) Show use of the separation of variables method to obtain a function for in terms of .

(5 marks)

(b) At what rate is the person losing weight after weeks? (2 marks)

(c) State the total weight that this person is expected to lose if they maintain the diet.

(1 mark)

Question 13 (8 marks)

A person who weighs kg begins a specialist diet so that their rate of weight loss can be modelled by

where is the persons weight in kilograms and is the number of days since the diet began.

After week the person had lost a total of kg.

(a) Show use of the separation of variables method to obtain a function for in terms of .

(5 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ separates variables  ü integrates both sides, including a constant  ü eliminates logs  ü obtains value of constant  ü obtains value of constant and writes function |

(b) At what rate is the person losing weight after weeks? (2 marks)

|  |
| --- |
| Solution |
| Losing weight at a rate of kg per day. |
| Specific behaviours |
| ✓ weight on correct day  ü correct rate of loss (allow sensible rounding) |

(c) State the total weight that this person is expected to lose if they maintain the diet.

(1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct weight loss |

Question 16 (7 marks)

A particle travels in a straight line so that its displacement cm at time seconds, relative to fixed point , satisfies the equation . Initially it has a displacement of cm and is moving away from . It moves with a period of seconds and an amplitude of cm.

(a) Determine a suitable function for . (3 marks)

(b) Determine the speed of the particle when it has a displacement of cm. (2 marks)

(c) Determine the distance travelled by the particle in the first seconds. (2 marks)

Question 16 (7 marks)

A particle travels in a straight line so that its displacement cm at time seconds, relative to fixed point , satisfies the equation . Initially it has a displacement of cm and is moving away from . It moves with a period of seconds and an amplitude of cm.

(a) Determine a suitable function for . (3 marks)

|  |
| --- |
| Solution |
| For SHM we can use [or .  Amplitude is and period is and so .  When . |
| Specific behaviours |
| ✓ indicates general form of solution  ü indicates amplitude and period  ü calculates phase shift and writes function |

(b) Determine the speed of the particle when it has a displacement of cm. (2 marks)

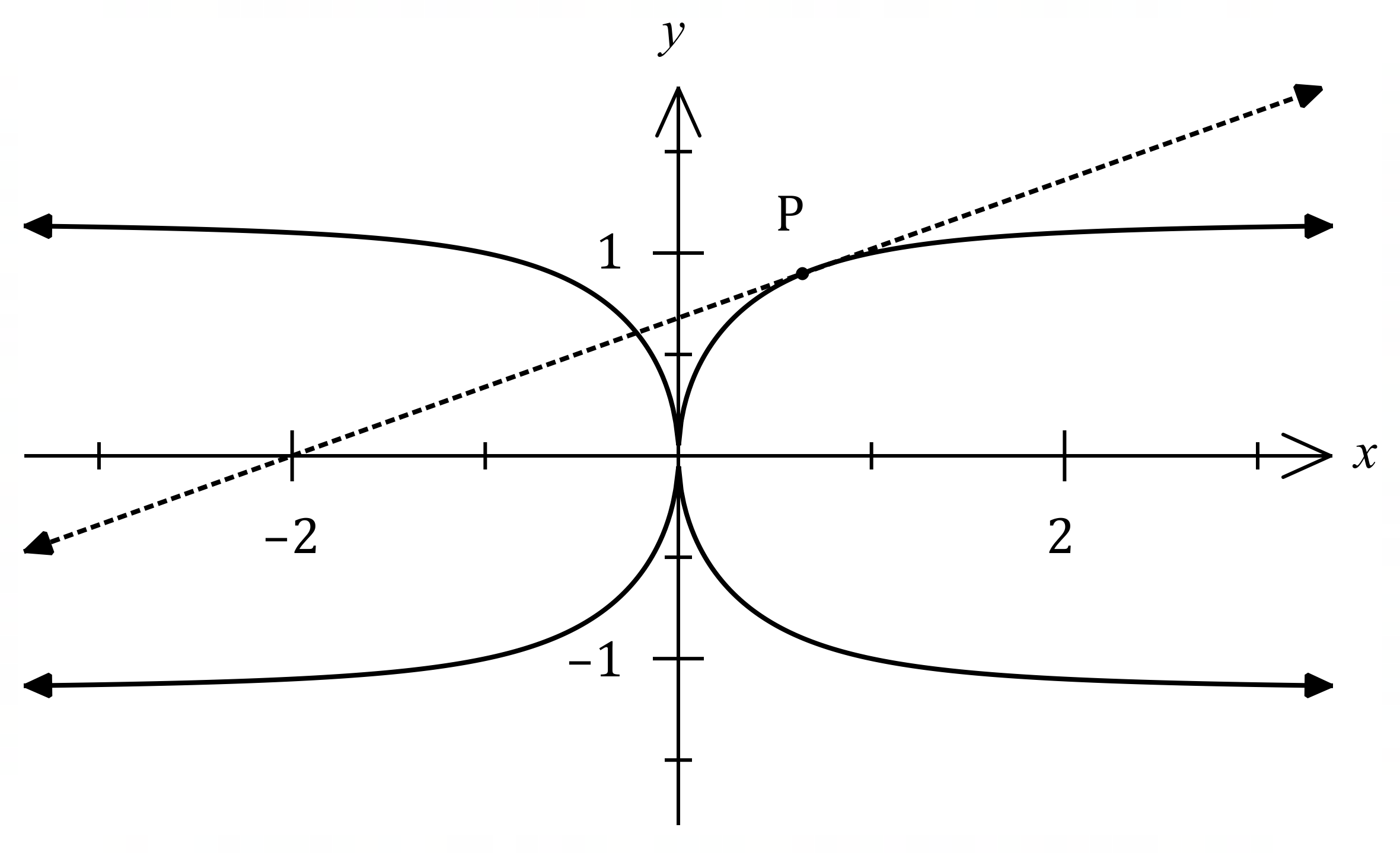
|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü substitutes  ü calculates speed |

(c) Determine the distance travelled by the particle in the first seconds. (2 marks)

|  |
| --- |
| Solution |
| Distance travelled in one cycle is cm.  Hence in cycle will travel cm. |
| Specific behaviours |
| ✓ indicates appropriate method  ü correct distance |

Question 21 (7 marks)

The graph of the relationship is shown below, together with the tangent to the curve at that passes through the point .



(a) Use implicit differentiation to obtain an expression for . (3 marks)

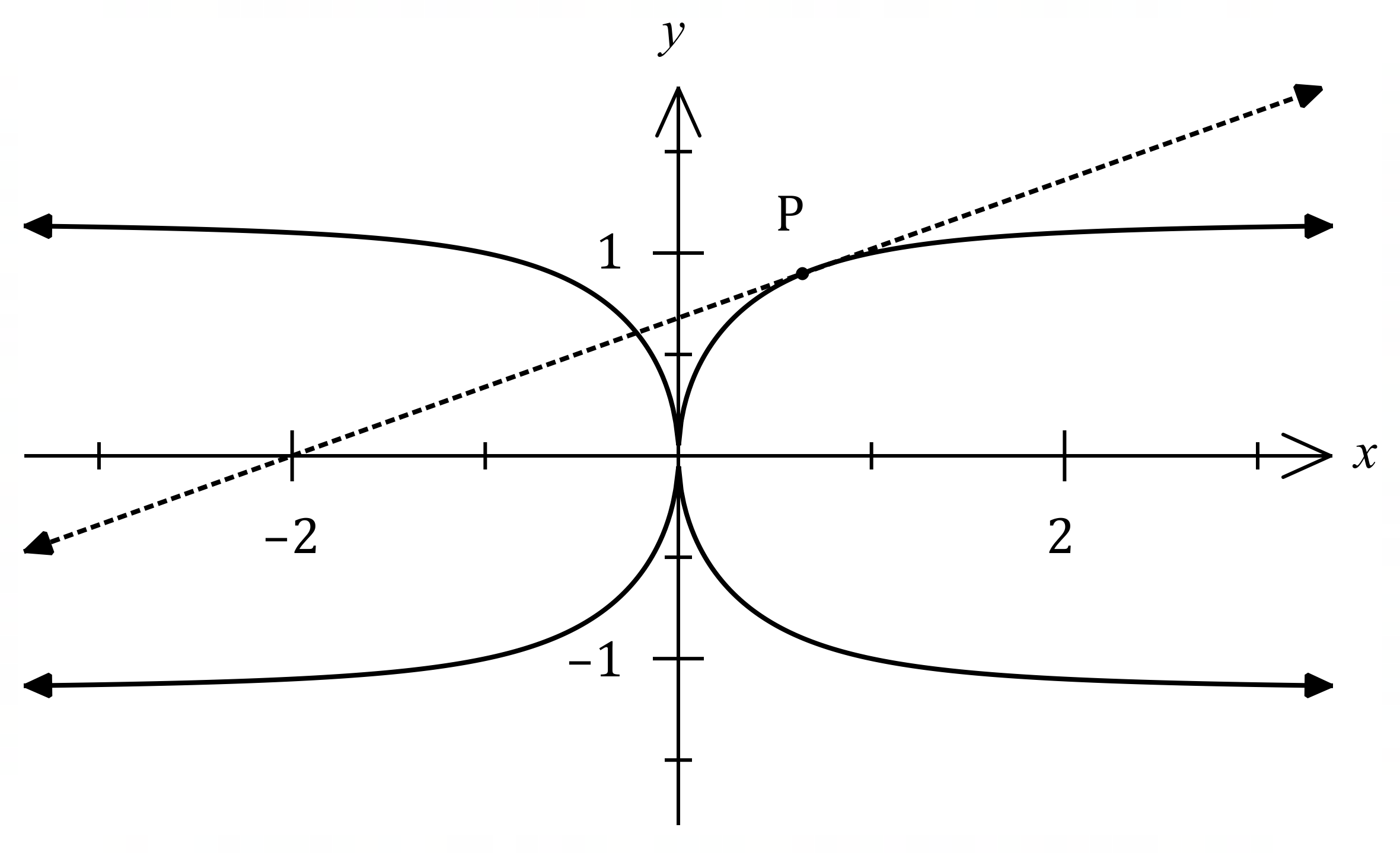
(b) Determine the slope of the curve at the point . (1 mark)

(c) Deduce that the -coordinate of is a solution to the equation .

(3 marks)

Question 21 (7 marks)

The graph of the relationship is shown below, together with the tangent to the curve at that passes through the point .



(a) Use implicit differentiation to obtain an expression for . (3 marks)

|  |
| --- |
| Solution |
| *NB May expand and then diff or diff and then expand* |
| Specific behaviours |
| ✓ differentiates product correctly  ü differentiates remainder correctly  ü rearranges for |

(b) Determine the slope of the curve at the point . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü correct value |

(c) Deduce that the -coordinate of is a solution to the equation .

(3 marks)

|  |
| --- |
| Solution |
| The equation of the tangent through is:  But from part (a), and so:  NB From given relationship, , and so:  Substitute for in relationship: |
| Specific behaviours |
| ✓ uses gradients to obtains equation (1)  ü obtains equation (2)  ü obtains equation (3) in one variable and simplifies |

Question 11 (7 marks)

The growth rate of electric vehicle (EV) sales as a percentage of all passenger vehicle (PV) sales in Australia can be modelled by

At the start of 2013 ( years), EV sales were of all PV sales in Australia. years later, had increased from to . The maximum expected percentage of EV sales is .

(a) Using a standard formula, or otherwise, express as a function of time . (3 marks)

(b) Determine

(i) the percentage of EV sales expected at the start of 2025. (1 mark)

(ii) the year in which EV sales are expected to reach of all PV sales. (1 mark)

(c) State the year in which the growth rate of EV sales as a percentage of PV sales will reach a maximum and determine this maximum growth rate. (2 marks)

Question 11 (7 marks)

The growth rate of electric vehicle (EV) sales as a percentage of all passenger vehicle (PV) sales in Australia can be modelled by

At the start of 2013 ( years), EV sales were of all PV sales in Australia. years later, had increased from to . The maximum expected percentage of EV sales is .

(a) Using a standard formula, or otherwise, express as a function of time . (3 marks)

|  |
| --- |
| **Alternative Solution** |
| Alt logistic equation, and :  Using :  Hence |
| **Specific behaviours** |
| ✓ uses logistic equation with and   solves for   correct equation |

|  |
| --- |
| **Solution** |
| Logistic equation with and :  Using :  Hence |
| **Specific behaviours** |
| ✓ uses logistic equation with and   solves for   correct equation |

(b) Determine

(i) the percentage of EV sales expected at the start of 2025. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct percentage |

(ii) the year in which EV sales are expected to reach of all PV sales. (1 mark)

|  |
| --- |
| **Solution** |
| . During 2031. |
| **Specific behaviours** |
| ✓ correct year |

(c) State the year in which the growth rate of EV sales as a percentage of PV sales will reach a maximum and determine this maximum growth rate. (2 marks)

|  |
| --- |
| **Solution** |
| Require . During 2028. |
| **Specific behaviours** |
| ✓ correct year   correct growth rate |

Question 21 (8 marks)

Particles and travel in a straight line with displacement m and velocity m/s at time s.

(a) The acceleration of is given by m/s2, and when , and .  
Determine, correct to one decimal place, the displacement of after seconds.

(5 marks)

(b) The acceleration of is given by . Determine, correct to two decimal places, the time taken for its velocity to increase from m/s to m/s. (3 marks)

Question 21 (8 marks)

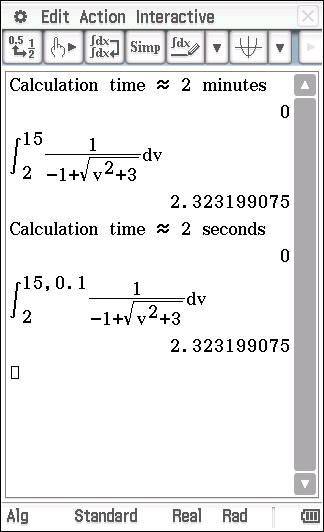
Particles and travel in a straight line with displacement m and velocity m/s at time s.

(a) The acceleration of is given by m/s2, and when , and .  
Determine, correct to one decimal place, the displacement of after seconds.

(5 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses and separates variables   expression for with constant   correct expression for   uses to obtain integral for   correct displacement |

(b) The acceleration of is given by . Determine, correct to two decimal places, the time taken for its velocity to increase from m/s to m/s. (3 marks)



|  |
| --- |
| **Solution** |
| Net change is integral of rate of change: |
| **Specific behaviours** |
| ✓ expression for   integral for net change   correct time |

Question 16 (7 marks)

Water, containing grams of dissolved sugar per litre, flows into a tank at a constant rate of  
 litres per hour.

Water is drawn from the tank, initially containing litres of water with no dissolved sugar, at the same constant rate of litres per hour.

Let the weight of sugar in the tank after hours be grams and assume that the sugar is always evenly dissolved throughout the water in the tank.

(a) By considering the rate at which dissolved sugar flows in and out of the tank, show that

(2 marks)

The water drawn from the tank can be used in a manufacturing process once the level of dissolved sugar exceeds grams per litre.

(b) Derive an equation for in terms of and hence determine how long this will take.

(5 marks)

Question 16 (7 marks)

Water, containing grams of dissolved sugar per litre, flows into a tank at a constant rate of  
 litres per hour.

Water is drawn from the tank, initially containing litres of water with no dissolved sugar, at the same constant rate of litres per hour.

Let the weight of sugar in the tank after hours be grams and assume that the sugar is always evenly dissolved throughout the water in the tank.

(a) By considering the rate at which dissolved sugar flows in and out of the tank, show that

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  derives rate out  ✓ derives rate in and uses difference |

The water drawn from the tank can be used in a manufacturing process once the level of dissolved sugar exceeds grams per litre.

(b) Derive an equation for in terms of and hence determine how long this will take.

(5 marks)

|  |
| --- |
| **Solution** |
| Will take hours mins. |
| **Specific behaviours** |
| ✓ separates variables   integrates both sides, shows constant   eliminates logs   correct equation   solves for |

Question 20 (9 marks)

Researchers used a simulation to model the population of foxes and the population of rabbits on an island. The rates of change of each population after years are given by

and .

(a) Briefly explain how the rabbit population is changing. (1 mark)

(b) Show that . (2 marks)

The equation in part (b) suggests that a model of the form would be appropriate, where and are positive constants.

(c) Explain this choice of model. (1 mark)

The research model used and the initial size of the fox population was .

(d) Determine the value of and the value of . (2 marks)

(e) Determine an equation for in terms of and use it to calculate the number of years until the rabbit population becomes extinct. (3 marks)

Question 20 (9 marks)

Researchers used a simulation to model the population of foxes and the population of rabbits on an island. The rates of change of each population after years are given by

and .

(a) Briefly explain how the rabbit population is changing. (1 mark)

|  |
| --- |
| **Solution** |
| Rabbit population is decreasing at a rate proportional to the population of foxes. |
| **Specific behaviours** |
| ✓ states decreasing |

(b) Show that . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ differentiates both sides   substitutes and simplifies |

The equation in part (b) suggests that a model of the form would be appropriate, where and are positive constants.

(c) Explain this choice of model. (1 mark)

|  |
| --- |
| **Solution** |
| The equation in (b) is of the form  and represents simple harmonic motion. |
| **Specific behaviours** |
| ✓ indicates SHM |

The research model used and the initial size of the fox population was .

(d) Determine the value of and the value of . (2 marks)

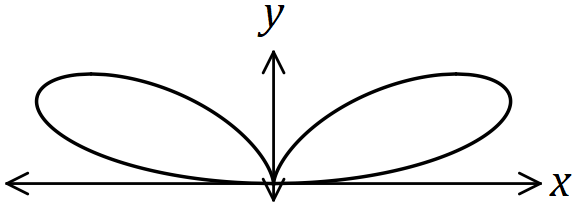
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ value of   value of |

(e) Determine an equation for in terms of and use it to calculate the number of years until the rabbit population becomes extinct. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ obtains   obtains   correct time |

Question 11 (7 marks)

(a) A bifolium has equation .



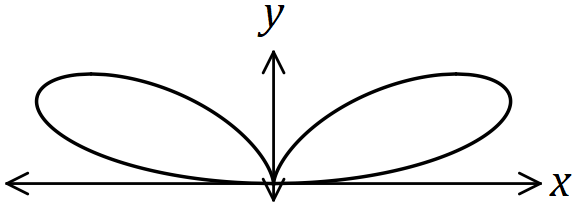
Show that the gradient of the bifolium at the point is . (4 marks)

(b) The gradient of a circle that passes through the point is given by

Determine the equation of the circle. (3 marks)

Question 11 (7 marks)

(a) A bifolium has equation .



Show that the gradient of the bifolium at the point is . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ implicit diff of RHS   implicit diff of LHS   substitutes   simplifies |

(b) The gradient of a circle that passes through the point is given by

Determine the equation of the circle. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ separates variables   integrates   correct equation, no specific form required |

Question 17 (7 marks)

A company recently introduced a new electronic control device for homes. In one city, the number of households , in thousands, that own the device months after observations began can be modelled by

(a) Use the model to determine

(i) the maximum number of households expected to own the device. (1 mark)

(ii) how long it will take for the number of households owning the device to double from the initial number. (2 marks)

(b) Show that the rate of change of the population satisfies the equation and determine the value of the constant . (4 marks)

Question 17 (7 marks)

A company recently introduced a new electronic control device for homes. In one city, the number of households , in thousands, that own the device months after observations began can be modelled by

(a) Use the model to determine

(i) the maximum number of households expected to own the device. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(ii) how long it will take for the number of households owning the device to double from the initial number. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ initial number   correct time |

(b) Show that the rate of change of the population satisfies the equation and determine the value of the constant . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct derivative of   substitutes for denominator of   systematic simplification   value of |

Question 20 (7 marks)

A particle moves with velocity in a straight line so that its acceleration is given by

Distances are measured in metres and times are in seconds. Initially the particle is at the origin () and has velocity .

(a) Use to express the velocity of the particle as a function of its displacement .

(6 marks)

(b) Determine the exact distance of the particle from the origin when its velocity .

(1 mark)

Question 20 (7 marks)

A particle moves with velocity in a straight line so that its acceleration is given by

Distances are measured in metres and times are in seconds. Initially the particle is at the origin () and has velocity .

(a) Use to express the velocity of the particle as a function of its displacement .

(6 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses required form of acceleration   separates variables   integrates   writes in exponential form (*attn to removal of absolute value*)   determines constant   correct equation |

(b) Determine the exact distance of the particle from the origin when its velocity .

(1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct distance |

Question 15 (6 marks)

A body moves in a straight line, so that at any time seconds its displacement, in metres, from a fixed point on the line is given by

(a) The body passes every seconds. Determine . (2 marks)

(b) Determine the speed at which the body passes . (2 marks)

(c) Determine the acceleration of the body when m. (2 marks)

Question 15 (6 marks)

A body moves in a straight line, so that at any time seconds its displacement, in metres, from a fixed point on the line is given by

(a) The body passes every seconds. Determine . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates period  ✓ states |

(b) Determine the speed at which the body passes . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ substitutes correctly into SHM equation  ✓ states speed |

(c) Determine the acceleration of the body when m. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates that  ✓ states acceleration |